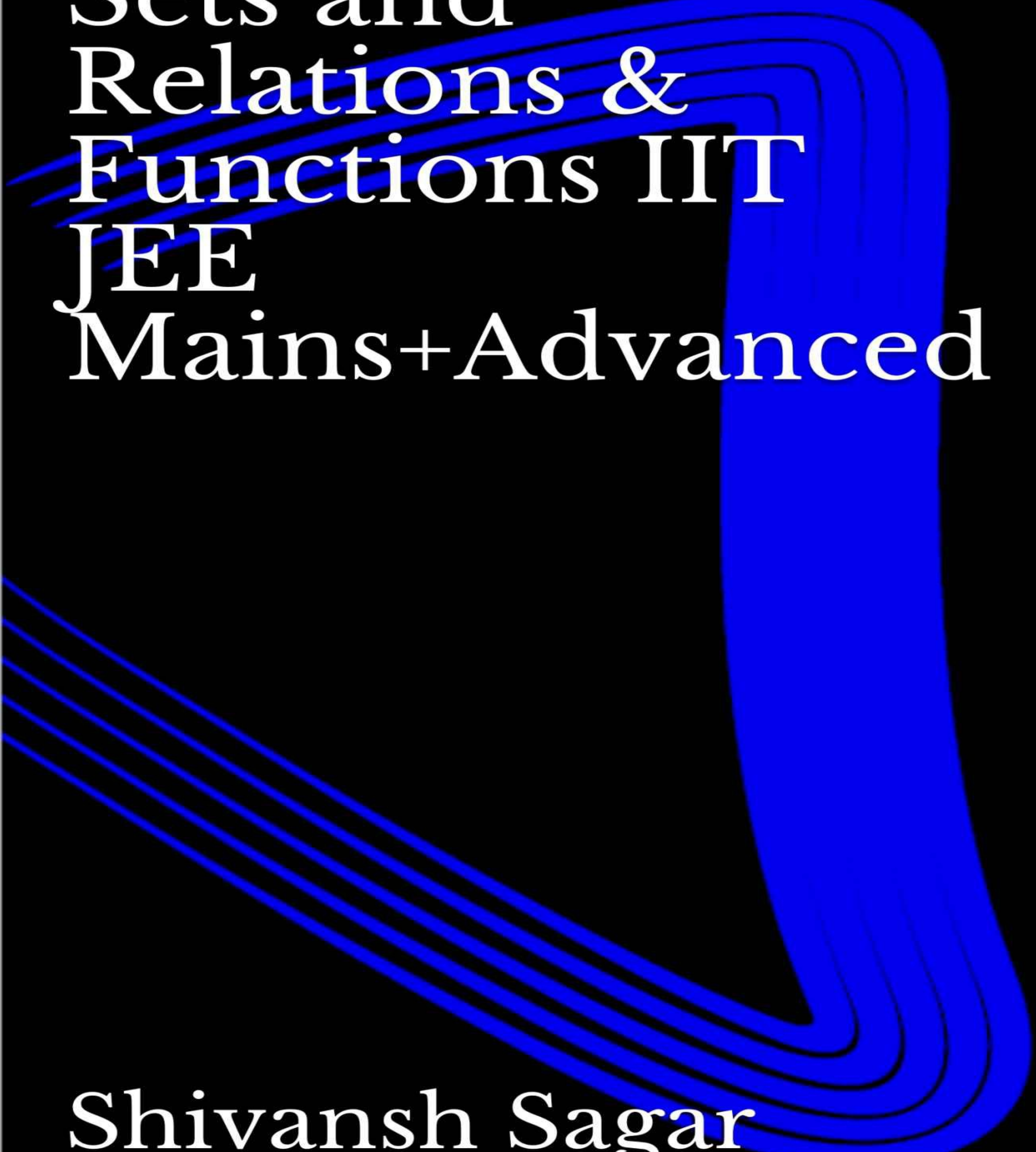
Sets and Relations & Functions IIT JEE Mains+Advanced



Shivansh Sagar

Sets and Relations

SETS

SET

A set is a collection of well defined objects which are distinct from each other. Set are generally denoted by capital letters A, B, C, ... etc. and the elements of the set by small letters a, b, c .... etc.

If a is an element of a set A, then we write $a \in A$ and say a belongs to A.

If a does not belong to A then we write $a \notin A$,

e.g. the collection of first five prime natural numbers is a set containing the elements 2, 3, 5, 7, 11.

METHODS TO WRITE A SET :

(i) Roster Method or Tabular Method : In this method a set is described by listing elements, separated by commas and enclose then by curly brackets. Note that while writing the set in roster form, an element is not generally repeated e.g. the set of letters of word SCHOOL may be written as {S, C, H, O, L}.

(ii) Set builder form (Property Method) : In this we write down a property or rule which gives us all the element of the set.

$A = \{x : P(x)\}$ where $P(x)$ is the property by which $x \in A$ and colon (:) stands for 'such that'

Example # 1 : Express set $A = \{x : x \in N \text{ and } x = 2n \text{ for } n \in N\}$ in roster form

Solution : $A = \{2, 4, 6, ...\}$

Example # 2 : Express set $B = \{x^2 : x < 4, x \in W\}$ in roster form

Solution : $B = \{0, 1, 4, 9\}$

Example # 3 : Express set $A = \{2, 5, 10, 17, 26\}$ in set builder form

Solution : $A = \{x : x = n^2 + 1, n \in N, 1 \le n \le 5\}$

TYPES OF SETS

Null set or empty set : A set having no element in it is called an empty set or a null set or void set, it is denoted by ϕ or { }. A set consisting of at least one element is called a non-empty set or a non-void set.

Singleton set : A set consisting of a single element is called a singleton set.

Finite set : A set which has only finite number of elements is called a finite set.

Order of a finite set : The number of elements in a finite set A is called the order of this set and denoted by O(A) or n(A). It is also called cardinal number of the set.

e.g. $A = \{a, b, c, d\} \Rightarrow n(A) = 4$

Infinite set : A set which has an infinite number of elements is called an infinite set.

Equal sets : Two sets A and B are said to be equal if every element of A is member of B, and every element of B is a member of A. If sets A and B are equal, we write $A = B$ and if A and B are not equal then $A \ne B$

Equivalent sets : Two finite sets A and B are equivalent if their number of elements are same i.e. $n(A) = n(B)$

e.g. $A = \{1, 3, 5, 7\}$, $B = \{a, b, c, d\} \Rightarrow n(A) = 4$ and $n(B) = 4$

$\Rightarrow$ A and B are equivalent sets

Note - Equal sets are always equivalent but equivalent sets may not be equal

Example # 4 : Identify the type of set :

(i) $A = \{x \in N : 5 < x < 6\}$ (ii) $A = \{a, b, c\}$

(iii) $A = \{1, 2, 3, 4, ...\}$ (iv) $A = \{1, 2, 6, 7\}$ and $B = \{6, 1, 2, 7, 7\}$

(v) $A = \{0\}$

Solution :
(i) Null set
(ii) finite set
(iii) infinite set
(iv) equal sets
(v) singleton set

Self Practice Problem :

(1) Write the set of all integers 'x' such that $|x - 3| < 8$.
(2) Write the set {1, 2, 3, 6} in set builder form.
(3) If $A = \{x : |x| < 2, x \in Z\}$ and $B = \{-1, 1\}$ then find whether sets A and B are equal or not.

Answers
(1) [–4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
(2) {x : x is a natural number and a divisor of 6}
(3) Not equal sets

SUBSET AND SUPERSET :

Let A and B be two sets. If every element of A is an element B then A is called a subset of B and B is called superset of A. We write it as $A \subseteq B$.

e.g. $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6, 7\}$ $\Rightarrow$ $A \subseteq B$

If A is not a subset of B then we write $A \not\subseteq B$

PROPER SUBSET :

If A is a subset of B but $A \neq B$ then A is a proper subset of B and we write $A \subset B$. Set A is not proper subset of A so this is improper subset of A

Note : (i) Every set is a subset of itself

(ii) Empty set ϕ is a subset of every set

(iii) $A \subseteq B$ and $B \subseteq \mathbf{A} \Leftrightarrow A = B$

(iv) The total number of subsets of a finite set containing n elements is 2^n.
(v) Number of proper subsets of a set having n elements is $2^n - 1$.
(vi) Empty set ϕ is proper subset of every set except itself.

POWER SET :

Let A be any set. The set of all subsets of A is called power set of A and is denoted by P(A)

Example # 5 : Examine whether the following statements are true or false :

(i) $\{a, b\} \not\subseteq \{b, c, a\}$
(ii) $\{a, e\} \not\subseteq \{x : x \text{ is a vowel in the English alphabet}\}$
(iii) $\{1, 2, 3\} \subseteq \{1, 3, 5\}$
(iv) $\{a\} \in \{a, b, c\}$

Solution :
(i) False as {a, b} is subset of {b, c, a}
(ii) True as a, e are vowels
(iii) False as element 2 is not in the set {1, 3, 5}
(iv) False as $a \in \{a, b, c\}$ and $\{a\} \subseteq \{a, b, c\}$

Example # 6 : Find power set of set $A = \{1, 2\}$

Solution : $P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

Example # 7 : If ϕ denotes null set then find $P(P(P(\phi)))$

Solution : Let $P(\phi) = \{\phi\}$

$P(P(\phi)) = \{\phi, \{\phi\}\}$

$P(P(P(\phi))) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$

Self Practice Problem :

(4) State true/false : $A = \{1, 3, 4, 5\}$, $B = \{1, 3, 5\}$ then $A \subseteq B$.

(5) State true/false : $A = \{1, 3, 7, 5\}$, $B = \{1, 3, 5, 7\}$ then $A \subset B$.

(6) State true/false : $[3, 7] \subseteq (2, 10)$

Answers (4) False (5) False (6) True

UNIVERSAL SET :

A set consisting of all possible elements which occur in the discussion is called a universal set and is denoted by U.

e.g. if $A = \{1, 2, 3\}$, $B = \{2, 4, 5, 6\}$, $C = \{1, 3, 5, 7\}$ then $U = \{1, 2, 3, 4, 5, 6, 7\}$ can be taken as the universal set.

SOME OPERATION ON SETS :

(i) Union of two sets : $A \cup B = \{x : x \in A \text{ or } x \in B\}$

e.g. $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ then $A \cup B = \{1, 2, 3, 4\}$

(ii) Intersection of two sets : $A \cap B = \{x : x \in A \text{ and } x \in B\}$

e.g. $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ then $A \cap B = \{2, 3\}$

(iii) Difference of two sets : $A - B = \{x : x \in A \text{ and } x \notin B\}$. It is also written as $A \cap B'$.

Similarly $B - A = B \cap A'$ e.g. $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$; $A - B = \{1\}$

(iv) Symmetric difference of sets : It is denoted by $A \Delta B$ and $A \Delta B = (A - B) \cup (B - A)$

(v) Complement of a set : $A' = \{x : x \notin A \text{ but } x \in U\} = U - A$

e.g. $U = \{1, 2, ..., 10\}$, $A = \{1, 2, 3, 4, 5\}$ then $A' = \{6, 7, 8, 9, 10\}$

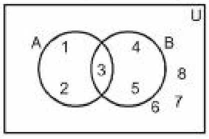
(vi) Disjoint sets : If $A \cap B = \phi$, then A, B are disjoint

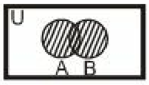
e.g. If $A = \{1, 2, 3\}$, $B = \{7, 8, 9\}$ then $A \cap B = \phi$

VENN DIAGRAM :

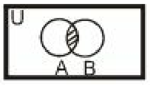
Most of the relationships between sets can be represented by means of diagrams which are known as venn diagrams. These diagrams consist of a rectangle for universal set and circles in the rectangle for subsets of universal set. The elements of the sets are written in respective circles.

For example If $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ then their venn diagram is

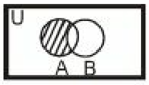




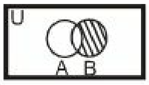
$A \cup B$



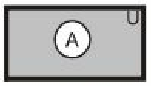
$A \cap B$



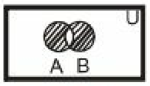
$A - B$



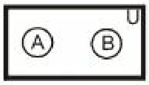
$B - A$



A'



$(A \Delta B) = (A - B) \cup (B - A)$



Disjoint

LAWS OF ALGEBRA OF SETS (PROPERTIES OF SETS):

(i) Commutative law : $(A \cup B) = B \cup A$; $A \cap B = B \cap A$

(ii) Associative law : $(A \cup B) \cup C = A \cup (B \cup C)$; $(A \cap B) \cap C = A \cap (B \cap C)$

(iii) Distributive law : $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(iv) De-morgan law : $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$

(v) Identity law : $A \cap U = A$; $A \cup \phi = A$

(vi) Complement law : $A \cup A' = U$, $A \cap A' = \phi$, $(A')' = A$

(vii) Idempotent law : $A \cap A = A$, $A \cup A = A$

NOTE :

(i) $A - (B \cup C) = (A - B) \cap (A - C)$; $A - (B \cap C) = (A - B) \cup (A - C)$

(ii) $A \cap \phi = \phi$, $A \cup U = U$

Example # 8 : Let $A = \{2, 4, 6, 8\}$ and $B = \{6, 8, 10, 12\}$ then find $A \cup B$

Solution : $A \cup B = \{2, 4, 6, 8, 10, 12\}$

Example # 9 : Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8\}$. Find $A - B$ and $B - A$.

Solution : $A - B = \{x : x \in A \text{ and } x \notin B\} = \{1, 3, 5\}$

similarly $B - A = \{8\}$

Example # 10 : State true or false :

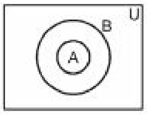
(i) $A \cup A' = \phi$ (ii) $\phi' \cap A = A$

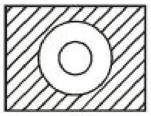
Solution : (i) false because $A \cup A' = U$

(ii) true as $\phi' \cap A = U \cap A = A$

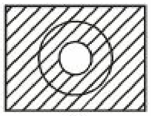
Example # 11 : Use Venn diagram to prove that $A \subseteq B \Rightarrow B' \subseteq A'$.

Solution :





B'



A'

From venn diagram we can conclude that $B' \subseteq A'$.

Example # 12 : Prove that if $A \cup B = C$ and $A \cap B = \phi$ then $A = C - B$.

Solution : Let $x \in A$ $\Rightarrow x \in A \cup B$ $\Rightarrow x \in C$ $(\because A \cup B = C)$

Now $A \cap B = \phi$ $\Rightarrow x \notin B$ $(\because x \in A)$

$\Rightarrow$ $x \in C - B$ $(\because x \in C \text{ and } x \notin B)$

$\Rightarrow$ $A \subseteq C - B$

Let $x \in C - B$ $\Rightarrow x \in C$ and $x \notin B$

$\Rightarrow$ $x \in A \cup B$ and $x \notin B$ $\Rightarrow x \in A$ $\Rightarrow$ $C - B \subseteq A$

$\therefore$ $A = C - B$

Self Practice Problem :

(7) Find $A \cup B$ if $A = \{x : x = 2n + 1, n \le 5, n \in N\}$ and $B = \{x : x = 3n - 2, n \le 4, n \in N\}$.

(8) Find A – (A – B) if A = {5, 9, 13, 17, 21} and B = {3, 6, 9, 12, 15, 18, 21, 24}

Answers (7) {1, 3, 4, 5, 7, 9, 10, 11} (8) {9, 21}

SOME IMPORTANT RESULTS ON NUMBER OF ELEMENTS IN SETS :

If A, B, C are finite sets and U be the finite universal set then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(iii) $n(A - B) = n(A) - n(A \cap B)$

(v) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(vi) Number of elements in exactly two of the sets A, B, C
$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

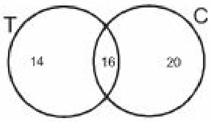
(vii) Number of elements in exactly one of the sets A, B, C
$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

Example # 13 : In a group of 40 students, 26 take tea, 18 take coffee and 8 take neither of the two. How many take both tea and coffee ?

Solution $n(U) = 40$, $n(T) = 26$, $n(C) = 18$
$n(T' \cap C') = 8 \Rightarrow n(T \cup C)' = 8$
$\Rightarrow n(U) - n(T \cup C) = 8$
$\Rightarrow n(T \cup C) = 32$
$\Rightarrow n(T) + n(C) - n(T \cap C) = 32$
$\Rightarrow n(T \cap C) = 12$

Example # 14 : In a group of 50 persons, 14 drink tea but not coffee and 30 drink tea. Find
(i) How many drink tea and coffee both ? (ii) How many drink coffee but not tea ?

Solution T : people drinking tea
C : people drinking coffee
(i) $n(T) = n(T - C) + n(T \cap C) \Rightarrow 30 = 14 + n(T \cap C) \Rightarrow n(T \cap C) = 16$



(ii) $n(C - T) = n(T \cup C) - n(T) = 50 - 30 = 20$

Self Practice Problem :

(9) Let A and B be two finite sets such that $n(A - B) = 15$, $n(A \cup B) = 90$, $n(A \cap B) = 30$. Find n(B)

(10) A market research group conducted a survey of 1000 consumers and reported that 720 consumers liked product A and 450 consumers liked product B. What is the least number that must have liked both products ?

Answers (9) 75 (10) 170

RELATIONS

ORDERED PAIR :

A pair of objects listed in a specific order is called an ordered pair. It is written by listing the two objects in specific order separating them by a comma abd enclosing the pair in parantheses.
In the ordered pair (a, b), a is called the first element and b is called the second element.

Two ordered pairs are set to be equal if their corresponding elements are equal. i.e.

$(a, b) = (c, d)$ if $a = c$ and $b = d$.

CARTESIAN PRODUCT :

The set of all possible ordered pairs (a, b), where $a \in A$ and $b \in B$ i.e. $\{(a, b) ; a \in A \text{ and } b \in B\}$ is called the cartesian product of A to B and is denoted by $A \times B$. Usually $A \times B \neq B \times A$.

Similarly $A \times B \times C = \{(a, b, c) : a \in A, b \in B, c \in C\}$ is called ordered triplet.

RELATION:

Let A and B be two sets. Then a relation R from A to B is a subset of $A \times B$. Thus, R is a relation from A to B $\Rightarrow R \subseteq A \times B$. The subsets is derived by describing a relationship between the first element and the second element of ordered pairs in $A \times B$ e.g. if $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $B = \{1, 2, 3, 4, 5\}$ and $R = \{(a, b) : a = b^2, a \in A, b \in B\}$ then $R = \{(1, 1), (4, 2), (9, 3)\}$. Here $a\,R\,b \Rightarrow 1\,R\,1, 4\,R\,2, 9\,R\,3$.

NOTE :

(i) Let A and B be two non-empty finite sets consisting of m and n elements respectively. Then $A \times B$ consists of mn ordered pairs. So total number of subsets of $A \times B$ i.e. number of relations from A to B is 2^{mn}.

(ii) A relation R from A to A is called a relation on A.

DOMAIN AND RANGE OF A RELATION :

Let R be a relation from a set A to a set B. Then the set of all first components of coordinates of the ordered pairs belonging to R is called to domain of R, while the set of all second components of corrdinates of the ordered pairs in R is called the range of R.

Thus, $\text{Dom}(R) = \{a : (a, b) \in R\}$ and $\text{Range}(R) = \{b : (a, b) \in R\}$

It is evident from the definition that the domain of a relation from A to B is a subset of A and its range is a subset of B.

Example # 1 : If $A = \{1, 2\}$ and $B = \{3, 4\}$, then find $A \times B$.

Solution : $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$

Example # 2 : Let $A = \{1, 3, 5, 7\}$ and $B = \{2, 4, 6, 8\}$ be two sets and let R be a relation from A to B defined by the phrase "$(x, y) \in R \Rightarrow x > y$". Find relation R and its domain and range.

Solution : Under relation R, we have 3R2, 5R2, 5R4, 7R4 and 7R6

i.e. $R = \{(3, 2), (5, 2), (5, 4), (7, 2), (7, 4), (7, 6)\}$

$\therefore$ $\text{Dom}(R) = \{3, 5, 7\}$ and $\text{range}(R) = \{2, 4, 6\}$

Example # 3 : Let $A = \{2, 3, 4, 5, 6, 7, 8, 9\}$. Let R be the relation on A defined by $\{(x, y) : x \in A, y \in A \text{ and } x \text{ divides } y\}$.
Find domain and range of R.

Solution: The relation R is
$R = \{(2, 2), (2, 4), (2, 6), (2, 8), (3, 3), (3, 6), (3, 9), (4, 4), (4, 8), (5, 5), (6, 6), (7, 7), (8, 8), (9, 9)\}$

Domain of $R = \{2, 3, 4, 5, 6, 7, 8, 9\} = A$
Range of $R = \{2, 3, 4, 5, 6, 7, 8, 9\} = A$

Self Practice Problem :

(1) If $(2x + y, 7) = (5, y - 3)$ then find x and y.

(2) If $A \times B = \{(1, 2), (1, 3), (1, 6), (7, 2), (7, 3), (7, 6)\}$ then find sets A and B.

(3) If A = {x, y, z} and B = {1, 2} then find number of relations from A to B.

(4) Write $R = \{(4x + 3, 1 - x) : x \le 2, x \in N\}$

Answers (1) $x = -\frac{5}{2}, y = 10$

(2) A = {1, 7}, B = {2, 3, 6}

(3) 64

(4) {(7, 0), (11, –1)}

TYPES OF RELATIONS :

In this section we intend to define various types of relations on a given set A.

(i) **Void relation :** Let A be a set. Then $\phi \subseteq A \times A$ and so it is a relation on A. This relation is called the void or empty relation on A.

(ii) **Universal relation :** Let A be a set. Then $A \times A \subseteq A \times A$ and so it is a relation on A. This relation is called the universal relation on A.

(iii) **Identity relation :** Let A be a set. Then the relation $I_A = \{(a, a) : a \in A\}$ on A is called the identity relation on A. In other words, a relation I_A on A is called the identity relation if every element of A is related to itself only.

(iv) **Reflexive relation :** A relation R on a set A is said to be reflexive if every element of A is related to itself. Thus, R on a set A is not reflexive if there exists an element $a \in A$ such that $(a, a) \notin R$.

Note : Every identity relation is reflexive but every reflexive relation in not identity.

(v) **Symmetric relation :** A relation R on a set A is said to be a symmetric relation iff $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$. i.e. $a R b \Rightarrow b R a$ for all $a, b \in A$.

(vi) **Transitive relation :** Let A be any set. A relation R on A is said to be a transitive relation iff $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$ i.e. $a R b$ and $b R c \Rightarrow a R c$ for all $a, b, c \in A$

(vii) **Equivalence relation :** A relation R on a set A is said to be an equivalence relation on A iff

(i) it is reflexive i.e. $(a, a) \in R$ for all $a \in A$

(ii) it is symmetric i.e. $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$

(iii) it is transitive i.e. $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b \in A$

Example # 4 : Which of the following are identity relations on set A = {1, 2, 3}.
$R_1 = \{(1, 1), (2, 2)\}$, $R_2 = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$, $R_3 = \{(1, 1), (2, 2), (3, 3)\}$.

Solution: The relation R_3 is idenity relation on set A.
R_1 is not identity relation on set A as $(3, 3) \notin R_1$.
R_2 is not identity relation on set A as $(1, 3) \in R_2$

Example # 5 : Which of the following are reflexive relations on set A = {1, 2, 3}.
$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 3), (2, 1)\}$, $R_2 = \{(1, 1), (3, 3), (2, 1), (3, 2)\}$..

Solution : R_1 is a reflexive relation on set A.
R_2 is not a reflexive relation on A because $2 \in A$ but $(2, 2) \notin R_2$.

Example # 6 : Prove that on the set N of natural numbers, the relation R defined by $x R y \Rightarrow x$ is less than y is transitive.

Solution : Because for any $x, y, z \in N$ $x < y$ and $y < z \Rightarrow x < z \Rightarrow x R y$ and $y R z \Rightarrow x R z$. so R is transitive.

Example # 7 : Let T be the set of all triangles in a plane with R a relation in T given by $R = \{(T_1, T_2) : T_1$ is congruent to $T_2\}$. Show that R is an equivalence relation.

Solution : Since a relation R in T is said to be an equivalenece relation if R is reflexive, symmetric and transitive.

(i) Since every triangle is congruent to itself
$\therefore$ R is reflexive
(ii) $(T_1, T_2) \in R \Rightarrow T_1$ is congruent to T_2
$\Rightarrow T_2$ is congruent to T_1
$\Rightarrow (T_2, T_1) \in R$
Hence R is symmetric
(iii) Let $(T_1, T_2) \in R$ and $(T_2, T_3) \in R$
$\Rightarrow T_1$ is congruent to T_2
and T_2 is congruent to T_3
$\Rightarrow T_1$ is congruent to T_3
$\Rightarrow (T_1, T_3) \in R$
$\therefore$ R is transitive
Hence R is an equivalence relation.

Example # 8 : Show that the relation R in R defined as $R = \{(a, b) : a \le b\}$ is transitive.

Solution : Let $(a, b) \in R$ and $(b, c) \in R$
$\therefore$ $(a \le b)$ and $b \le c \Rightarrow a \le c$
$\therefore$ $(a, c) \in R$
Hence R is transitive.

Example # 9 : Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is symmetric.

Solution : Let $(a, b) \in R$ $[\because (1, 2) \in R]$
$\therefore$ $(b, a) \in R$ $[\because (2, 1) \in R]$
Hence R is symmetric.

Self Practice Problem :

(5) Let L be the set of all lines in a plane and let R be a relation defined on L by the rule $(x, y) \in R$ $\Rightarrow$ x is perpendicular to y. Then prove that R is a symmetric relation on L.

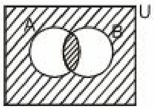
(6) Let R be a relation on the set of all lines in a plane defined by $(\ell_1, \ell_2) \in R \Rightarrow$ line ℓ_1 is parallel to line ℓ_2. Prove that R is an equivalence relation.

Board Level Exercise

Type (I) : Very Short Answer Type Questions : **[01 Mark Each]**

1. If U = Number of students in class XI of school S and A = Number of girls in class XI of school S. Find A′.

2. Represent the shaded part of the Venn diagram in terms of union and intersection of the sets A and B and their complements.



3. If the ordered pair $(x - 1, -5) = (2, y + 3)$ find x and y.

4. If $A \times B = \{(a, 1), (a, 2), (a, 5), (b, 2), (b, 5), (b, 1)\}$ then find $B \times A$.

5. If $A \times B = \{(a, 1), (b, 3), (a, 3), (b, 1), (a, 2), (b, 2)\}$ then find A and B.

6. Let $n(A) = n$. Then find the number of all relations on A.

7. If R is a relation from a finite set A having m elements to a finite set B having n elements, then find the number of relations from A to B.

8. Show that the relation R in R defined as $R = \{(a, b) : a \le b\}$ is not symmetric.

9. Show that the relation R in the set {1, 2, 3}, given by $R = \{(1, 2), (2, 1)\}$ is not reflexive.

Type (II) : Short Answer Type Questions : **[02 Marks Each]**

10. If $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 3, 5, 8\}$, $C = \{2, 5, 7, 8\}$ verify that $A - (B \cup C) = (A - B) \cap (A - C)$.

11. Given a universal set $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
Find the complement of each of the following :
(i) $A = \{1, 3, 7, 9\}$ (ii) $B = \{0\}$ (iii) $C = \phi$ (iv) $D = \{3, 7, 8, 9\}$

12. If $U = \{1, 2, 3, 4, \ldots\ldots, 10\}$ is the universal set for the sets $A = \{2, 3, 4, 5\}$ and $B = \{1, 2, 3, 4, 5, 6\}$ then verify that $(A \cup B)' = A' \cap B'$.

13. If $U = \{a, e, i, o, u\}$, $A = \{a, e, i\}$, $B = \{e, o, u\}$ and $C = \{a, i, u\}$ then verify that
$A \cap (B - C) = (A \cap B) - (A \cap C)$

14. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$. Verify that $(A \cap B)' = A' \cup B'$.

15. In a group of 40 students, 26 take tea, 18 take coffee and 8 take neither of the two. How many take both tea and coffee ?

16. List all the subsets of $\{-1, 0, 1\}$.

17. If $A = \{1, 3, 5, 6\}$ and $B = \{2, 4\}$ find $A \times B$ and $B \times A$.

18. Let $R = \{(x, y) : x, y \in N \text{ and } 2x + y = 9\}$, N being the set of all natural numbers. Write R as the set of ordered pairs.

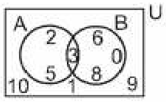
19. Let $R : A \to B$, where $A = \{3, 5\}$ and $B = \{7, 11\}$ and $R = \{(a, b) \in A \times B \mid a - b \text{ is an odd number}\}$. Write the relation R.

Type (III) : Long Answer Type Questions : [04 Mark Each]

20. Write the set of all the possible subsets (power set) of the set :
(i) $A = \{a, b\}$ (ii) $B = \{a, b, c\}$

21. If $A = \{a, b, c\}$; $B = \{d\}$, $C = \{e\}$ verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

22. From the adjoining Venn diagram, determine the following sets :



(i) $A \cup B$ (ii) $A \cap C$ (iii) $A - B$ (iv) $(A \cap B)'$

23. In a group of 50 persons, 14 drink tea but not coffee and 30 drink tea. Find
(i) How many drink tea and coffee both ? (ii) How many drink coffee but not tea ?

24. A and B are two sets such that $n(A - B) = 14 + x$, $n(B - A) = 3x$ and $n(A \cap B) = x$. Draw a Venn diagram to illustrate information and if $n(A) = n(B)$ then find the value of x.

25. If $P = \{a, b\}$ find $P \times P \times P$.

26. A relation R is defined from a set $A = \{2, 3, 4, 5\}$ to a set $B = \{3, 6, 7, 10\}$ as follows : $(x, y) \in R \Leftrightarrow x$ divides y. Express R as a set of ordered pairs and determine the domain and range of R.

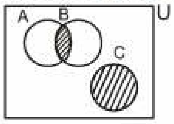
27. If R is relation 'is greater than' from $A = \{2, 3, 4, 5, 6\}$ to $B = \{2, 5, 6\}$ write the elements of R.

28. Check whether the relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

Type (IV) : Very Long Answer Type Questions: [06 Mark Each]

29. Show that the relation R in the set (1, 2, 3) given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is reflexive but neither symmetric nor transitive.

30. Express the shaded region of the adjoining Venn diagram in terms of union and intersection of the sets A, B and C :
Also if $n(U) = 50$, $n(A) = 26$
$n(C) = 12$, $n(A \cap B) = 20$ and
$n(A' \cap B') = 27$, find $n(B)$ and $n(A' \cap B' \cap C')$



31. Let $U = \{x \in N : x \le 8\}$, $A = \{x \in N : 5 < x^2 < 50\}$ and $B = \{x \in N : x \text{ is prime number less than } 10\}$. Draw a Venn diagram to show the relationship between the given sets. Hence list the elements of the following sets (i) A' (ii) B' (iii) $A - B$ (iv) $A \cap B'$ (v) is $A - B = A \cap B'$?

32. In a survey of 25 students, it was found that 15 had taken Mathematics, 12 had Physics and 11 had taken Chemistry, 5 had taken Mathematics and Chemistry, 9 had taken Mathematics and Physics, 4 had taken Physics and Chemistry and 3 had taken all the three subjects. Find the number of students that had taken :
(A) Only Chemistry (B) Only Mathematics (C) Only one of the subjects

33. A survey of 500 television viewers produced the given information : 285 watch football, 195 watch hockey, 115 watch cricket, 45 watch football and cricket, 70 watch football and hockey, 50 watch cricket and hockey, 50 do not watch any of the three games. How many watch exactly one of the three games ?

34. In a class of 35 students, 17 have taken Mathematics, 10 have taken Mathematics but not Economics. Find the number of students who have taken both Mathematics and Economics and the number of students who have taken Economics but not Mathematics, if it is given that each student has taken either Mathematics or Economics or both.

35. Let $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{2, 4, 6\}$, $B = \{3, 5\}$ and $C = \{1, 2, 4, 7\}$. Find
(i) $A \cup B$ (ii) $B \cap C$ (iii) $A \cup (B \cap C)$ (iv) $(A \cap B) \cup C$ (v) $A' \cap B'$
(vi) $A \cap (B \cup C)'$ (vii) $A' \cup (B \cap C')$ (viii) $A - B$ (ix) $C - A$ (x) $(B - A) \cup (A - C)$

36. If $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$ then which of the following are relations from A to B ?
(i) $R_1 = \{(1, a), (1, b), (1, c)\}$ (ii) $R_2 = \{(1, c), (2, b), (2, a)\}$ (iii) $R_3 = \{(b, 2), (1, c), (3, a)\}$
(iv) $R_4 = \{(2, c), (3, c), (4, c)\}$ (v) $R_5 = \{(1, d), (2, c), (3, a)\}$

Exercise # 1

SUBJECTIVE QUESTIONS

Section (A) : Representation of set, Types of sets, Subset, Power Set.

A-1. Write the set of all vowels in English alphabet which precede letter O.

A-2. Classify the following as a finite or infinite set :
(i) $A = \{x \in N : (x - 1)(x - 2) = 0\}$ (ii) $B = \{x \in N : x \text{ is odd}\}$

A-3. Write the following set by roster method : The set of all natural numbers 'x' such that $4x + 9 < 50$.

A-4. Describe the following set by set property method $\{0, 3, 8, 15, 24, 35\}$

A-5. Describe the following set by roster method the set of all letters in the word TRIGONOMETRY.

A-6. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Find the values of m and n.

A-7. Which of the following are true ?
(i) If $A = \{1, 5, 5, 5\}$, $B = \{1, 3, 5\}$, then $A \not\subset B$.
(ii) If $A = \{x : x^3 - 1 = 0, x \in N\}$, $B = \{x : x^2 - 4x + 3 = 0, x \in N\}$ then $A \subseteq B$.

A-8. Assume that $P(A) = P(B)$. Prove that $A = B$.

Section (B) : Venn diagrams, Algebra of sets.

B-1. If $A = \{x : x = 4n + 1, n \le 5, n \in N\}$ and $B \{3n : n \le 8, n \in N\}$, then find $A - (A - B)$.

B-2. Prove that $A \cup B = A \cap B$ iff $A = B$.

B-3. Prove that : $A - (B \cup C) = (A - B) \cap (A - C)$ without using venn diagram.

B-4. Prove by using venn diagram
(i) $A - (B \cup C) = (A - B) \cap (A - C)$ (ii) $A \subseteq B \Rightarrow B' \subseteq A'$

Section (C) : Theorems on cardinal number

C-1. A and B are two sets such that $n(A) = 3$ and $n(B) = 6$.
Find (i) minimum value of $n(A \cup B)$ (ii) maximum value of $n(A \cup B)$

C-2. Of the members of three athletic teams in a school 21 are in the cricket team, 26 are in the hockey team and 29 are in the football team. Among them, 14 play hockey and cricket, 15 play hockey and football, and 12 play football and cricket. Eight play all the three games. Find the total number of members in the three athletic teams.

C-3. In a class of 55 students, the number of students studying different subjects are 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all the three subjects. Find the number of students who have taken exactly one subject.

Section (D) : Cartesian Product of Sets, Domain, Range and Co-domain of Relation.

D-1. Determine the domain and the range of the relation R defined by $R = \{(x + 1, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}\}$.

D-2. If $A = \{3, 4, 6\}$, $B = \{1, 3\}$ and $C = \{1, 2, 6\}$ then find $(A - B) \times (A - C)$.

Section (E) : Types of Relations

E-1. Let n be a fixed positive integer. Define a relation R on the set of integers Z, $aRb \Leftrightarrow n|(a - b)$. Then prove that R is equivalence relation

E-2. Let R be a relation over the set $N \times N$ and it is defined by $(a, b)\ R\ (c, d) \Rightarrow a + d = b + c$. Then prove that R is equivalence relation

E-3. Let L be the set of all straight lines in the Euclidean plane. Two lines ℓ_1 and ℓ_2 are said to be related by the relation R if ℓ_1 is parallel to ℓ_2. Then prove that R is equivalence relation.

E-4. For $n, m \in N$, $n \mid m$ means that n is a factor of m, then prove that relation $|$ is reflexive, transitive but not symmetric.

E-5. Let $R = \{(x, y) : x, y \in A, x + y = 5\}$ where $A = \{1, 2, 3, 4, 5\}$ then prove that R is neither reflexive nor transitive but symmetric.

Exercise # 2

PART - I : OBJECTIVE QUESTIONS

*** Marked Questions may have more than one correct option.**

Section (A) : Representation of set, Types of sets, Subset, Power Set.

A-1. The set of intelligent students in a class is-
(A) a null set
(B) a singleton set
(C) a finite set
(D) not a well defined collection

A-2. Which of the following is the empty set
(A) $\{x : x \text{ is a real number and } x^2 - 1 = 0\}$
(B) $\{x : x \text{ is a real number and } x^2 + 1 = 0\}$
(C) $\{x : x \text{ is a real number and } x^2 - 9 = 0\}$
(D) $\{x : x \text{ is a real number and } x^2 = x + 2\}$

A-3. The set $A = \{x : x \in R, x^2 = 16 \text{ and } 2x = 6\}$ is
(A) Null set (B) Singleton set (C) Infinite set (D) None of these

A-4. If $A = \{x : |x| < 3, x \in Z\}$ then the number of subsets of A is -
(A) 120 (B) 30 (C) 31 (D) 32

A-5. Which of the following are true ?
(A) $[3, 7] \subseteq (2, 10)$ (B) $(0, \infty) \subseteq (4, \infty)$ (C) $(5, 7] \subseteq [5, 7)$ (D) $[2, 7] \subseteq (2.9, 8)$

A-6. The number of subsets of the power set of set $A = \{7, 10, 11\}$ is
(A) 32 (B) 16 (C) 64 (D) 256

Section (B) : Venn diagrams, Algebra of sets.

B-1. Sets A and B have 3 and 6 elements respectively. What can be the minimum number of elements in $A \cup B$?
(A) 3 (B) 6 (C) 9 (D) 18

B-2. Given the sets $A = \{1, 2, 3\}$, $B = \{3, 4\}$, $C = \{4, 5, 6\}$, then $A \cup (B \cap C)$ is
(A) $\{3\}$ (B) $\{1, 2, 3, 4\}$ (C) $\{1, 2, 4, 5\}$ (D) $\{1, 2, 3, 4, 5, 6\}$

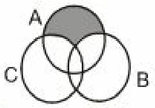
B-3. Let $A = \{x : x \in R, |x| < 1\}$, $B = \{x : x \in R, |x - 1| \geq 1\}$ and $A \cup B = R - D$, then the set D is
(A) $\{x : 1 < x \leq 2\}$ (B) $\{x : 1 \leq x < 2\}$ (C) $\{x : 1 \leq x \leq 2\}$ (D) None of these

B-4. The smallest set A such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$ is
(A) $\{2, 3, 5\}$ (B) $\{3, 5, 9\}$ (C) $\{1, 2, 5, 9\}$ (D) None of these

B-5. Let A and B be two sets. Then
(A) $A \cup B \leq A \cap B$ (B) $A \cap B \leq A \cup B$ (C) $A \cap B = A \cup B$ (D) None of these

B-6. If $A = \{2, 3, 4, 8, 10\}$, $B = \{3, 4, 5, 10, 12\}$, $C = \{4, 5, 6, 12, 14\}$ then $(A \cap B) \cup (A \cap C)$ is equal to
(A) $\{3, 4, 10\}$ (B) $\{2, 8, 10\}$ (C) $\{4, 5, 6\}$ (D) $\{3, 5, 14\}$

B-7. The shaded region in the given figure is



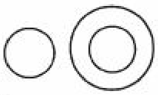
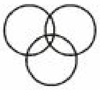
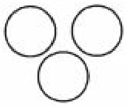
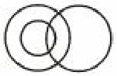
(A) $A \cap (B \cup C)$ (B) $A \cup (B \cap C)$ (C) $A \cap (B - C)$ (D) $A - (B \cup C)$

B-8. Let $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$, then $n(A' \cap B') =$
(A) 400 (B) 600 (C) 300 (D) 200

B-9. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 5\}$, $B = \{6, 7\}$, then $A \cap B'$ is
(A) B' (B) A (C) A' (D) B

B-10. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n - 1) ; n \in N\}$, then $X \cup Y$ is equal to
(A) X (B) Y (C) N (D) None of these

B-11. Which of the following venn-diagrams best represents the sets of females, mothers and doctors ?

(A)

(B)

(C)

(D)


Section (C) : Theorems on cardinal number

C-1. In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspaper is-
(A) at least 30 (B) at most 20 (C) exactly 25 (D) none of these

C-2. In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3 % buy B and C and 4% buy A and C. If 2% families buy all the three news papers, then number of families which buy newspaper A only is
(A) 3100 (B) 3300 (C) 2900 (D) 1400

C-3. In a city 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then persons travelling by car or bus is
(A) 80 percent (B) 40 percent (C) 60 percent (D) 70 percent

C-4. A class has 175 students. The following data shows the number of students obtaining one or more subjects : Mathematics 100, Physics 70, Chemistry 40, Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23, Mathematics & Physics & Chemistry 18. How many students have offered Mathematics alone ?
(A) 35 (B) 48 (C) 60 (D) 22

Section (D) : Cartesian Product of sets, Domain, range and co-domain of Relation.

D-1. If A = {a, b}, B = {c, d}, C = {d, e}, then {(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)} is equal to
(A) $A \cap (B \cup C)$ (B) $A \cup (B \cap C)$ (C) $A \times (B \cup C)$ (D) $A \times (B \cap C)$

D-2. If A = {2, 4, 5}, B = {7, 8, 9}, then $n(A \times B)$ is equal to
(A) 6 (B) 9 (C) 3 (D) 0

D-3. If $A = \{x : x^2 - 5x + 6 = 0\}$, B = {2, 4}, C = {4, 5} then $A \times (B \cap C)$ is-
(A) {(2, 4), (3, 4)} (B) {(4, 2), (4, 3)} (C) {(2, 4), (3, 4), (4, 4)} (D) {(2, 2), (3, 3), (4, 4), (5, 5)}

D-4. Let A = {a, b, c} and B = {1, 2}. Consider a relation R defined from set A to set B. Then R is equal to set
(A) A (B) B (C) $A \times B$ (D) $B \times A$

D-5. A and B are two sets having 3 and 4 elements respectively and having 2 elements in common. The number of relation which can be defined from A to B is
(A) 2^5 (B) $2^{10} - 1$ (C) $2^{12} - 1$ (D) none of these

D-6. Let R be relation from a set A to a set B, then
(A) $R = A \cup B$ (B) $R = A \cap B$ (C) $R \subseteq A \times B$ (D) $R \subseteq B \times A$

D-7. Let X = {1, 2, 3, 4, 5} and Y = {1, 3, 5, 7, 9}. Which of the following is not a relation from X to Y
(A) $R_1 = \{(x, y) \mid y = 2 + x, x \in X, y \in Y\}$ (B) $R_2 = \{(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)\}$
(C) $R_3 = \{(1, 1), (1, 3) (3, 5), (3, 7), (5, 7)\}$ (D) $R_4 = \{(1, 3), (2, 5), (2, 4), (7, 9)\}$

Section (E) : Types of Relations

E-1. Let A = {1, 2, 3, 4} and R be a relation in A given
by R = {(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 1), (1, 3)}, then relation R is
(A) Reflexive (B) Symmetric (C) Equivalence (D) Reflexive and Symmetric

E-2. The relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)} on set A = {1, 2, 3} is
(A) Reflexive but not symmetric (B) Reflexive but not transitive
(C) Symmetric and Transitive (D) Neither symmetric nor transitive

E-3. The relation "less than" in the set of natural number is
(A) Only symmetric (B) Only transitive (C) Only reflexive (D) Equivalence relation

E-4. The relation R defined in N as aRb $\Leftrightarrow$ b is divisible by a is
(A) Reflexive but not symmetric (B) Symmetric but not transitive
(C) Symmetric and transitive (D) None of these

E-5. In the set A = {1, 2, 3, 4, 5} a relation R is defined by R = $\{(x, y) | x, y \in A \text{ and } x < y\}$. Then R is
(A) Reflexive (B) Symmetric (C) Transitive (D) None of these

E-6. For real numbers x and y, we write $x\ R\ y \Rightarrow x - y + \sqrt{2}$ is an irrational number. Then the relation R is-
(A) Reflexive (B) Symmetric (C) Transitive (D) None of these

E-7. Which one of the following relations on R is equivalence relation-
(A) $x\ R_1 y \Leftrightarrow |x| = |y|$ (B) $x\ R_2 y \Leftrightarrow x \geq y$
(C) $x\ R_3 y \Leftrightarrow x \mid y$ (x divides y) (D) $x\ R_4 y \Leftrightarrow x < y$

E-8. The relation R defined in A = {1, 2, 3} by a R b if $|a^2 - b^2| \leq 5$. Which of the following is false-
(A) R = {(1, 2), (2, 2), (3, 3), (2, 1), (2, 3), (3, 2)} (B) Co-domain of R = {1, 2, 3}
(C) Domain of R = {1, 2, 3} (D) Range of R = {1, 2, 3}

E-9. Let P = $\{(x, y) \mid x^2 + y^2 = 1, x, y \in R\}$, then P is
(A) reflexive (B) symmetric (C) transitive (D) equilance

E-10. Let A = {p, q , r}. Which of the following is an equivalence relation on A ?
(A) R_1 = {(p, q), (q, r), (p, r) (p, p)} (B) R_2 = {(r, q), (r, p), (r, r) (q, q)}
(C) R_3 = {(p, p), (q, q), (r, r) (p, q)} (D) none of these

E-11. Let R_1 be a relation defined by $R_1 = \{(a, b) \mid a \geq b ; a, b \in R\}$. Then R_1 is
(A) An equivalence relation on R (B) Reflexive, transitive but not symmetric
(C) Symmetric, Transitive but not reflexive (D) Neither transitive nor reflexive but symmetric

E-12. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha R \beta \Leftrightarrow \alpha \perp \beta, \alpha, \beta \in L$. The R is
(A) Reflexive (B) Symmetric (C) Transitive (D) None of these

E-13. Let S be the set of all real numbers. Then the relation R = {(a, b) : 1 + ab > 0} on S is
(A) Reflexive and symmetric but not transitive (B) Reflexive, transitive but not symmetric
(C) Symmetric, transitive but not reflexive (D) Reflexive, transitive and symmetric

E-14. Let R be a relation on the set N be defined by $\{(x, y) | x, y \in N, 2x + y = 41\}$. Then R is
(A) Reflexive (B) Symmetric (C) Transitive (D) None of these

PART - II : COMPREHENSION

Comprehension # 1 (Q. 1 to 3)

In a group of 1000 people, there are 750 people, who can speak Hindi and 400 people, who can speak Bengali.

1. Number of people who can speak Hindi only is
(A) 300 (B) 400 (C) 500 (D) 600

2. Number of people who can speak Bengali only is
(A) 150 (B) 250 (C) 50 (D) 100

3. Number of people who can speak both Hindi and Bengali is
(A) 50 (B) 100 (C) 150 (D) 200

Comprehension # 2 (Q. 4 to 6)

Let R be a relation defined as $R = \{ (x, y) : y = |x - 1|, x \in Z \text{ and } |x| \le 3\}$

4. Relation R is equal to :
(A) $\{(1, 0), (1, 2), (3, 2), (4, 3)\}$
(B) $\{(-3, 4), (-2, 3), (-1, 2), (0, 1), (1, 0), (2, 1), (3, 2)\}$
(C) $\{(4, -3), (3, -2), (2\ -1), (1, 0), (2, 3)\}$
(D) None of these

5. Domain of R is :
(A) $\{0, 1, 2, 3, 4\}$
(B) $\{1, 3, 4\}$
(C) $\{-3, -2, -1, 0, 1, 2, 3\}$
(D) $\{0, 1, 2, 3, 4\}$

6. Range of R is
(A) $\{0, 1, 2, 3, 4\}$
(B) $\{-3, -2, -1, 0, 1, 2, 3\}$
(C) $\{-4, -3, -1, -2, 0\}$
(D) $\{-1, 0, 1, 2, 3, 4\}$

Exercise # 3

PART - I : AIEEE PROBLEMS (PREVIOUS YEARS)

1. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is-
(1) transitive (2) not symmetric (3) reflexive (4) a function **[AIEEE-2004]**

2. Let $R = \{(3, 3), (6, 6), (9, 9), (12, 12)\ (6, 12), (3, 9), (3, 12), (3, 6)\}$ be relation on the set $A = \{3, 6, 9, 12\}$. Then the relation R is **[AIEEE - 2005]**
(1) reflexive and transitive only
(2) reflexive only
(3) an equilvalence relation
(4) reflexive and symmetric only

3. Let W denote the words in the english dictionary. Define the relation R by : $R = \{(x, y) \in W \times W \mid$ the words x and y have at least one letter in common$\}$. Then R is- **[AIEEE - 2006]**
(1) reflexive, symmetric and not transitive
(2) reflexive, symmetric and transitive
(3) reflexive, not symmetric and transitive
(4) not reflexive, symmetric and transitive

4. Let R be the real line. Consider the following subsets of the plane R × R **[AIEEE-2008]**

$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$

$T = \{(x, y) : x - y \text{ is an integer}\}$

Which one of the following is true ?
(1) T is an equivalence relation on R but S is not
(2) Neither S nor T is an equivalence relation on R
(3) Both S and T are equivalence relations on R
(4) S is an equivalence relation on R but T is not

5. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then **[AIEEE-2009]**
(1) $A = C$ (2) $B = C$ (3) $A \cap B = \phi$ (4) $A = B$

6. Consider the following relations : **[AIEEE-2010]**
R : {(x, y)| x ,y are real numbers and x = wy for some rational number w}

S = $\left\{\left(\frac{m}{n}, \frac{p}{q}\right)\right.$ | m, n, p and q are integers such that n, q $\neq$ 0 and qm = pn}

Then
(1) neither R nor S is an equivalence relation
(2) S is an equivalence relation but R is not an equivalence relation
(3) R and S both are equivalence relations
(4) R is an equivalence relation but S is not an equivalence relation

7. Let R be the set of real numbers. **[AIEEE-2011, I]**
Statement-1 : A = {(x, y) $\in$ R × R : y – x is an integer} is an equivalence relation on R.
Statement-2 : B = {(x, y) $\in$ R × R : x = αy for some rational number α} is an equivalence relation on R.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true;Statement-2 is **not** a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

8. Consider the following relation R on the set of real square matrices of order 3. **[AIEEE - 2011, II]**
R = {(A, B)|A = P^{-1} BP for some invertible matrix P}.
Statement -1 : R is equivalence relation.
Statement - 2 : For any two invertible 3 × 3 matrices M and N, $(MN)^{-1} = N^{-1}M^{-1}$.
(1) Statement-1 is true, statement-2 is a correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

9. Let X = {1, 2, 3, 4, 5}. The number of different ordered pairs (Y, Z) that can formed such that Y $\subseteq$ X, Z $\subseteq$ X and Y $\cap$ Z is empty, is : **[AIEEE-2012]**
(1) 5^2 (2) 3^5 (3) 2^5 (4) 5^3

10. Let A and B two sets containing 2 elements and 4 elements respectively. The number of subsets of A × B having 3 or more elements is **[AIEEE - 2013, (4, –¼),360]**
(1) 256 (2) 220 (3) 219 (4) 211

PART - II : CBSE PROBLEMS (PREVIOUS YEARS)

1. Show that the relation R defined by (a, b) R(c, d) $\Rightarrow$ a + d = b + c on the set N × N is an equivalence relation. **[CBSE - 2008]**

2. Prove that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is even}, is an equivalence relation. **[CBSE - 2009]**

3. Let Z be the set of all integers and R be the relation on Z defined as R = {(a, b) : a, b $\in$ Z, and (a – b) is divisible by 5} Prove that R is an equivalence relation. **[CBSE - 2010]**

4. Show that the relation S in the set R of real numbers, defined as S = {(a, b) : a, b $\in$ R and a $\leq b^3$} is neither reflexive, nor symmetric nor transitive. **[CBSE - 2010]**

5. Show that the relation S in the set A = {x $\in$ Z : 0 $\leq$ x $\leq$ 12} given by S = {(a, b) : a, b $\in$ Z, |a – b| is divisible by 4} is an equivalence relation. Find the set of all elements related to 1. **[CBSE - 2010]**

6. Show that the relation S defined on the set N × N by (a, b) S(c, d) $\Rightarrow$ a + d = b + c is an equivalence relation. **[CBSE - 2010]**

7. Let f : X $\rightarrow$ Y be a function. Define a relation R on X given by R = {(a, b) : f(a) = f(b)}. Show that R is an equivalence relation on X . **[CBSE - 2010]**

8. State the reason for the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} not to be transitive. **[CBSE - 2011]**

Answers

BOARD LEVEL SOLUTIONS

1. Boys of class XI + Girls of class XI = Students of class XI
$\therefore$ A′ = Not A = No. of boys in class XI of school S

2. $(A \cup B)' \cup (A \cap B)$ or $(A - B)' \cap (B - A)'$

3. $x - 1 = 2, -5 = y + 3$

4. $B \times A$ can be obtained from $A \times B$ by interchanging the entries in $A \times B$.
$\therefore B \times A = \{(1, a), (2, a), (5, a), (1, b), (2, b), (5, b)\}$

5. A is the set of first entries and B is the set of second entries
$\therefore A = \{a, b\}, B = \{1, 2, 3\}$

6. $n(A) = n$,No. of elements in $A \times A = n^2$

Total relations defined for $A \to A = 2^{n^2}$

7. Number of elements in $A \times B = mn$
$\Rightarrow$ Number of relations from A to B = 2^{mn}

8. Let $(a, b) \in R$
$\therefore a \le b \not\Rightarrow b \le a$
$\therefore (b, a) \notin R$
Hence R is not symmetric.

9. Since $(1, 1) \notin R$ also $(2, 2) \notin R$ and $(3, 3) \notin R$
$\therefore (a, a) \notin R$
Hence R is not reflexive.

10. $B \cup C = \{1, 2, 3 ,5, 7, 8\}$
$A - (B \cup C) = \{4\}$
$A - B = \{2, 4\}$
$A - C = \{1, 3, 4\}$
$(A - B) \cap (A - C) = \{4\}$

11. (i) $A' = \{0, 2, 4, 5, 6, 8\}$
(ii) $B' = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
(iii) $C' = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
(iv) $D' = \{0, 1, 2, 4, 5, 6\}$

12. $A \cup B = \{1, 2, 3, 4, 5, 6\}$
$(A \cup B)' = \{7, 8, 9, 10\}$
$A' = \{1, 6, 7, 8, 9, 10\}$
$B' = \{7, 8, 9, 10\}$
$A' \cap B' = \{7, 8, 9, 10\}$

13. $B - C = \{e, o\}$
$A \cap (B - C) = \{e\}$
$A \cap B = \{e\}$
$A \cap C = \{a, i\}$
$(A \cap B) - (A \cap C) = \{e\}$

14. $A \cap B = \{2\}$
$(A \cap B)' = \{1, 3, 4, 5, 6, 7, 8, 9\}$
$A' = \{1, 3, 5, 7, 9\}$
$B' = \{1, 4, 6, 8, 9\}$
$A' \cup B' = \{1, 3, 4, 5, 6, 7, 8, 9\}$

15. $n(U) = 40, n(T) = 26, \quad n(C) = 18$
$n(T' \cap C') = 8 \Rightarrow n(T \cup C)' = 8$
$\Rightarrow n(U) - n(T \cup C) = 8$
$\Rightarrow n(T \cup C) = 32$
$\Rightarrow n(T) + n(C) - n(T \cap C) = 32$
$\Rightarrow n(T \cap C) = 12$

16. The subsets are ϕ, $\{-1\}$, $\{0\}$, $\{1\}$, $\{-1, 0\}$, $\{-1, 1\}$, $\{0, 1\}$, $\{-1, 0, 1\}$

17. $A \times B = \{(1, 2), (1, 4), (3, 2), (3, 4), (5, 2), (5, 4), (6, 2), (6, 4)\}$
$B \times A = \{(2, 1), (2, 3), (2, 5), (2, 6), (4, 1), (4, 3), (4, 5), (5, 6)\}$

18. As $y = 9 - 2x$ so if $x = 1, y = 7$; $x = 2, y = 5$ etc.
$\therefore R = \{(1, 7), (2, 5), (3, 3), (4, 1)\}$

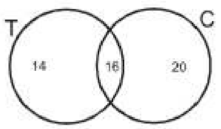
19. In set A and set B, elements are odd numbers so $a - b$ i.e. difference of two odd number is always even.
so relation R is empty relation.

20. (i) $P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$
(ii) $B = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}$

21. $B \cup C = \{d, e\}$
$A \times (B \cup C) = \{a, b, c\} \times \{d, e\} = \{(a, d), (a, e), (b, d), (b, e), (c, d), (c, e)\}$
$A \times B = \{a, b, c\} \times \{d\} = \{(a, d), (b, d), (c, d)\}$ ...(i)
$A \times C = \{a, b, c\} \times \{e\} = \{(a, e), (b, e), (c, e)\}$
$(A \times B) \cup (A \times C) = \{(a, d), (b, d), (c, d), (a, e), (b, e), (c, e)\}$ ...(ii)
from (i) and (ii), L.H.S. = R.H.S.

22. (i) $A \cup B = \{0, 2, 3, 5, 6, 8\}$
(ii) $A \cap C = \{3\}$
(iii) $A - B = \{2, 5\}$
(iv) $(A \cap B)' = \{0, 1, 2, 5, 6, 8, 9, 10, 12\}$

23. T : people drinking tea
C : people drinking coffee
(i) $n(T) = n(T - C) + n(T \cap C)$
$\Rightarrow 30 = 14 + n(T \cap C) \Rightarrow n(T \cap C) = 16$



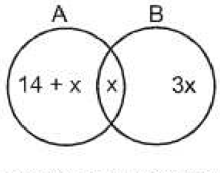
(ii) $n(C - T) = n(T \cup C) - n(T) = 50 - 30 = 20$

24. $n(A) = n(B)$

$\Rightarrow n(A - B) + n(A \cap B) = n(B - A) + n(A \cap B)$

$\Rightarrow 14 + x + x = 3x + x$

$\Rightarrow 14 = 2x \Rightarrow x = 7$



25. $P \times P \times P = \{a, b\} \times \{a, b\} \times \{a, b\} = \{(a, a), (a, b), (b, a), (b, b)\} \times \{a, b\}$

26. a|b stands for 'a divides b'. For the elements of the given sets A and B, we find that 2|6, 2|10, 3|3, 3|6 and 5|10

$\therefore (2, 6) \in R,, (2, 10) \in R, (3, 3) \in R, (3, 6) \in R$ and $(5, 10) \in R$

Thus, $R = \{(2, 6), (2, 10), (3, 3), (3, 6), (5, 10)\}$

Also Domain of $R = \{2, 3, 5\}$,

Range of $R = \{3, 6, 10\}$

27. If $x \in A$, $y \in B$ then $x > y$,

so $R = \{(3, 2), (4, 2), (5, 2), (6, 2), (6, 5)\}$

28. Here $R = \{(a, b) : b = a + 1\}$

$= \{(a, a + 1) : a, a + 1 \in \{1, 2, 3, 4, 5, 6\}\}$

$= \{(1, 2)\ (2, 3), (3, 4), (4, 5), (5, 6)\}$

(i) R is not reflexive as $(a, a) \notin R\ \forall\ a$.

(ii) R is not symmetric as $(a, b) \in R$ but $(b, a) \notin R$

(iii) R is not transitive as $(a, b) \in R$ and $(a, c) \in R$ but $(a, c) \notin R$

$\because$ $(1, 2) \in R$, $(2, 3) \in R$ but $(1, 3) \notin R$

29. Since (1, 1), (2, 2) and (3, 3) lie in R

$\therefore$ R is reflexive.

Also $(1, 2) \in R$ but $(2, 1) \notin R$

$\therefore$ R is not symmetric

since $(1, 2) \in R$ and $(2, 3) \in R$ but $(1, 3) \notin R$

$\therefore$ R is not transitive.

30. Shaded region is $(A \cap B) \cup C$

$n(A' \cap B') = 27 \Rightarrow n(A \cup B)' = 27$

$\Rightarrow n(U) - n(A \cup B) = 27$

$\Rightarrow n(A \cup B) = 50 - 27 = 23$

$\Rightarrow n(A) + n(B) - n(A \cap B) = 23$

$\Rightarrow n(B) = 23 - 26 + 20 = 17$

$n(A' \cap B' \cap C') = n(A \cup B \cup C)'$

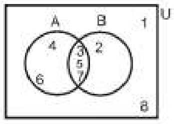
$= n(U) - n(A \cup B \cup C) = 50 - 23 - 12 = 15$

31. $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$A = \{3, 4, 5, 6, 7\}$; $B = \{2, 3, 5, 7\}$

$A' = \{1, 2, 8\}$

$B' = \{1, 4, 6, 8\}$



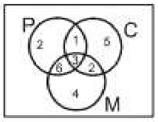
$A - B = \{x : x \in A \text{ and } x \notin B\} = \{4, 6\}$

$A \cap B' = \{3, 4, 5, 6, 7\} \cap \{1, 4, 6, 8\} = \{4, 6\}$

32. $n(P \cup C \cup M) = 25$

$n(M) = 15, n(P) = 12, n(C) = 11$

$n(M \cap C) = 5, n(M \cap P) = 9, n(P \cap C) = 4$



$n(P \cap C \cap M) = 3$

From Venn diagram n(only C) = 5,

n(only M) = 4, n(only P $\cup$ Only C $\cup$ Only M) = 11

33. $n(F \cup H \cup C) = n(U) - n(F \cup H \cup C)'$

$= 500 - 50 = 450$

$n(F \cup H \cup C) = n(F) + n(H) + n(C) - n(F \cap H)$

$- n(H \cap C) - n(C \cap F) + n(F \cap C \cap H)$

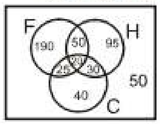
$\Rightarrow 450 = 285 + 195 + 115 - 70 - 50 - 45$

$+ n(F \cap C \cap H)$

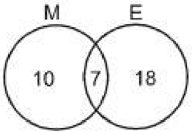
$\Rightarrow n(F \cap C \cap H) = 20$

From Venn diagram required number of viewers =

$190 + 95 + 40 = 325$



34. M : Mathematics, E : Economics



$n(M \cup E) = 35, n(M) = 17, n(M - E) = 10$

$n(M \cap E) = n(M) - n(M - E) = 17 - 10 = 7$

$n(E - M) = n(E \cup M) - n(M) = 35 - 17 = 18$

35. (i) $A \cup B = \{2, 3, 4, 5, 6\}$ (ii) $B \cap C = \phi$

(iii) {2, 4, 6} (iv) {1, 2, 4, 7} (v) {1, 7}

(vi) {6} (vii) {1, 3, 5, 7} (viii) {2, 4, 6}

(ix) {1, 7} (x) {3, 5, 6}

36. (i) relation, as domain of $R_1 \subset A$.

(ii) relation, as domain of $R_2 \subset A$.

(iii) not a relation, as domain of $R_3 \not\subset A$.

(iv) not a relation, as domain of $R_4 \not\subset A$.

(v) not a relation, as range of $R_5 \not\subset B$.

EXERCISE # 1

Section (A) :

A-1. {a, e, i}

A-2. (i) Finite (ii) Infinite

A-3. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

A-4. $\{x : x = n^2 - 1, n \in N, n \le 6\}$

A-5. {T, R, I, G, O, N, M, E, Y}

A-6. n = 3, m = 6 **A-7.** (i) False (ii) True

Section (B) :

B-1. {9, 21}

Section (C) :

C-1. (i) 6 (ii) 9 **C-2.** 43 **C-3.** 22

Section (D) :

D-1. Domain = {1, 2, 3, 4, 5, 6},
Range = {5, 6, 7, 8, 9, 10}

D-2. {(4, 3), (4, 4), (6, 3), (6, 4)}

EXERCISE # 2

PART - I

Section (A) :

A-1. (D) **A-2.** (B) **A-3.** (A)

A-4. (D) **A-5.** (A) **A-6.** (D)

Section (B) :

B-1. (B) **B-2.** (B) **B-3.** (B)

B-4. (B) **B-5.** (B) **B-6.** (A)

B-7. (D) **B-8.** (C) **B-9.** (B)

B-10. (B) **B-11.** (D)

Section (C) :

C-1. (C) **C-2.** (B) **C-3.** (C)

C-4. (C)

Section (D) :

D-1. (C) **D-2.** (B) **D-3.** (A)

D-4. (C) **D-5.** (D) **D-6.** (C)

D-7. (D)

Section (E) :

E-1. (D) **E-2.** (A) **E-3.** (B)

E-4. (A) **E-5.** (C) **E-6.** (A)

E-7. (A) **E-8.** (A) **E-9.** (B)

E-10. (D) **E-11.** (B) **E-12.** (B)

E-13. (A) **E-14.** (D)

PART - II

1. (D) **2.** (B) **3.** (C) **4.** (B) **5.** (C) **6.** (A)

EXERCISE # 3

PART - I

1. (2) **2.** (1) **3.** (1) **4.** (1) **5.** (2) **6.** (2)

7. (3) **8.** (2) **9.** (2) **10.** (3)

PART - II

1. Reflexive
$\because a + b = b + a \Rightarrow (a, b) R (a, b)$
Hence R is reflexive.
(ii) Symmetric
$(a, b) R (c, d) \Rightarrow a + d = b + c$
$\Rightarrow b + c = a + d \Rightarrow c + b = d + a$
$\Rightarrow (c, d) R (a, b)$
Hence R is symmetric
(iii) $(a, b) R (c, d)$ and $(c, d) R (e, f)$
$\Rightarrow a + d = b + c$ and $c + f = d + e$
$\Rightarrow a + d + c + f = b + c + d + e$
$\Rightarrow a + f = b + e \Rightarrow (a, b) R (e, f)$
Hence R is transitive
Therefore, relation R is an equivalence relation.

2. **Reflexive :** Let $a \in A$
$\therefore |a - a| = 0$ is an even number
$\Rightarrow (a, a) \in R$
$\therefore$ R is reflexive
Symmetric : Let $a, b \in R$
Let $(a, b) \in R$
$\Rightarrow |a - b|$ is even $\Rightarrow |-(b - a)|$ is even
$\Rightarrow |b - a|$ is even
$\Rightarrow (b, a) \in R$
$\therefore$ R is symmetric
Transitive : Let $a, b, c \in R$
Let $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow |a - b|$ is even and $|b - c|$ is even
$\Rightarrow a - b$ is even and $b - c$ is even

$\Rightarrow (a - b) + (b - c)$ is even
$\Rightarrow |a - c|$ is even $\Rightarrow (a, c) \in R$
$\therefore$ R is transitive.
Hence R is an equivalence relation as R is reflexive, symmetric and transitive.

3. **Reflexive :** $\because a - a$ is divisible by 5 for all $a \in Z$
$\therefore$ R is reflexive
Symmetric : $(a, b) \in R \Rightarrow a - b$ is divisible by 5
$\Rightarrow b - a$ is divisible by 5
$\Rightarrow b - a \in R$
$\therefore$ R is symmetric
Transitive : $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow a - b$ and $b - c$ are both divisible by 5
$\Rightarrow a - b + b - c$ is divisible by 5
$\Rightarrow a - c$ is divisible by 5
$\Rightarrow (a, c) \in R$
$\therefore$ R is transitive
Since R is reflexive, symmetric and transitive.
Hence, R is an equivalence relation.

4. (i) As $a \le a^3$ is not true for all $a \in R$
$\therefore$ R is not reflexive
For example, if $a = \frac{1}{3}$ then $a > a^3$
i.e. $a \le a^3$ is not true
(ii) If $(a, b) \in R$ then need not imply that $(b, a) \in R$
$\therefore$ R is not symmetric
For example, if $(1, 2) \in R$ but $(2, 1) \notin R$,
As $1 \le 2^3$ but $2 \not\le 1^3$
(iii) If $(a, b) \in R$ and $(b, c) \in R$, then need not imply that $(a, c) \in R$
$\therefore$ R is not transitive
For example $(100, 5) \in R$ and $(5, 2) \in R$ but $(100, 2) \notin R$
As $100 \le 5^3$ and $5 \le 2^3$ but $100 \ge 2^3$.

5. **Reflexive :** For all $a \in A$
$|a - a| = 0$ is divisible by 4 $\Rightarrow (a, a) \in S$
$\therefore$ S is reflexive
Symmetric : Let $a, b \in A$, $(a , b) \in S$
$\Rightarrow |a - b|$ is divisible by 4
$\Rightarrow |b - a|$ is divisibe by 4 $\Rightarrow (b, a) \in S$
$\therefore$ S is symmetric
Transitive : Let $a, b, c \in A$, $(a, b) \in S$, $(b, c) \in S$
$\Rightarrow |a - b|$ is divisible by 4 and $|b - c|$ is divisible by 4
$\Rightarrow (a - b)$ and $(b - c)$ is divisible by 4
$\Rightarrow (a - b) + (b - c) = a - c$ is divisible by 4
$\Rightarrow |a - c|$ is divisible by 4 $\Rightarrow (a, c) \in S$
$\therefore$ S is transitive
Since S is reflexive, symmetric and transitive.
Hence, S is an equivalence relation.
The set of all elements of A, related to 1 is $\{1, 5, 9\}$

6. Reflexive
$\because a + b = b + a \Rightarrow (a, b) R(a, b)$
Hence R is reflexive.
(ii) Symmetric
$(a, b) R(c, d) \Rightarrow a + d = b + c$
$\Rightarrow b + c = a + d \Rightarrow c + b = d + a$
$\Rightarrow (c, d) R(a, b)$
Hence R is symmetric
(iii) $(a, b) R(c, d)$ and $(c, d) R(e, f)$
$\Rightarrow a + d = b + c$ and $c + f = d + e$
$\Rightarrow a + d + c + f = b + c + d + e$
$\Rightarrow a + f = b + e \Rightarrow (a, b) R(e, f)$
Hence R is transitive
Therefore, relation R is an equivalence relation.

7. **Reflexive :** For every $a \in X$, since $f(a) = f(a)$
$\therefore (a, a) \in R$ $\therefore$ R is reflexive
Symmetric : Let $(a, b) \in R$ for every $a, b \in X$
$\Rightarrow f(a) = f(b) \Rightarrow f(b) = f(a) \Rightarrow (b, a) \in R$
$\therefore$ R is symmetric
Transitive : Let $a, b, c \in X$. Let $(a, b) \in R$ and $(b, c) \in R \Rightarrow f(a) = f(b)$ and $f(b) = f(c)$
$\Rightarrow f(a) = f(c) \Rightarrow (a, c) \in R$
$\therefore$ R is transitive.
Since R is reflexive, symmetric and transitive.
Therefore R is an equivalence relation.

8. In the case of transitive relation
$(a, b), (b, c) \in R$
$\Rightarrow (a, c) \in R$
Here $(1, 2)$ and $(2, 1) \in R$ but $(1, 1) \notin R$.
So, it is not transitive.

Advanced Level Problems

SUBJECTIVE QUESTIONS

1. If A = {(x, y) : $x^2 + y^2 = 25$} and B = {(x, y) : $x^2 + 9y^2 = 144$} then find n(A ∩ B).

2. In a battle, 70% of the combatants lost one eye, 80% an ear, 75% an arm, 85% a leg, x% lost all the four limbs. Find the minimum value of x.

3. A survey shows that 63% of the Americans like cheese whereas 76% like apples. If x% of the Americans like both cheese and apples, find the value of x.

4. Prove that $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C' = B \cap C'$.

5. Let B be a subset of A and let P(A : B) = {S : B ⊆ S ⊆ A}. If A = {a, b, c, d} and B = {a, b}, then list all the elements of P(A : B).

6. Prove that the relation R = {(x, y) : $x^2 = xy$} is reflexive and transitive but not symmetric

7. Consider a relation R defined on set of square matrices A of order 2 satisfying $A^2 = I$. If A, B are two such matrices then relation R is defined as $(A, B) \in R \Rightarrow (AB)^T = A^TB^T$. Prove that relation R is reflexive and symmetric.

Answers

1. 4 **2.** 10 **3.** $x \in [39, 63]$ **5.** {a, b}, {a, b, c}, {a, b, d}, {a, b, c, d}